Classical Mechanics Module

1. What would be the duration of a day (now 24 hrs) if the Earth spun so fast such that bodies at its equator just floated? (JEST 2003; TIFR?)

   a) 24 h   b) 1.41 h
   c) 3.84 h   d) 12 h

2. What would be the mean distance from the sun in A.U. of an imaginary planet in the solar system whose period of revolution around the sun is 8 yrs. (1 AU = mean sun-earth distance) (JEST 2003)

   a) 8   b) 4
   c) 64   d) ~2.7

3. A homogenous spherical mass in hydrostatic equilibrium has radius R and mass density ρ. Pressure at a distance r from the centre is (G is the universal constant of gravitation)

   a) $2\pi\rho^2 G(R^2-r^2)/3$ b) ρG/3r
   c) ρG/3R   d) ρG/3

4. The planet Saturn is about 9 times more distant from the Sun than the planet Earth is. The period of revolution of Saturn around the Sun should be close to

   a) 27 years   b) 3 years
   c) 3 years   d) 81 years

5. A point particle of mass 'm' moves in the gravitational field of an infinite homogeneous mass distributed in the XY plane. For this test particle the following components of momentum P and angular momentum M are conserved: (JEST 2000) (Note that gravitational force acts in the z direction).

   a) P_z and M_z b) P_z, M_x and M_y
   c) P_x, P_y and M_z d) P_x, P_y, P_z & M_x, M_y, M_z

6. Masses 3m and 5m are attached to the two ends of a spring with spring constant 'k'. The period of oscillation is (JEST 2000)

   a) $2\pi\sqrt{\frac{8m}{15k}}$ b) $2\pi\sqrt{\frac{15m}{8k}}$

c) $2\pi\sqrt{\frac{2m}{15k}}$ d) $2\pi\sqrt{\frac{15m}{2k}}$

(reduced mass to be used)

7. Consider a rotating planet of radius 4000 km and acceleration due to gravity ¼ times that of Earth. What would be the rotation period (approximately) if the bodies on the equator were to become gravitationally unbound. (JEST 2000)

a) 4.42 hours b) 2.35 hours
c) 0.5 hours d) 1.12 hours

8. A homogeneous circular cylinder of length 'h', radius 'r' and mass 'm', rolls without slipping along an inclined plane in the Earth's gravitational field. If the velocity of the centre of mass is 'v' , its total kinetic energy is, (JEST 2001) (Note that total energy of the system ½ mv^2 + ½ I omega^2 where I=1/2 mr^2 and r omega=v, a condition for rolling without slipping).

a) ¼ mv^2 b) ½ mv^2
c) ¾ mv^2 d) mv^2

9. Consider the tides caused by the Moon on Earth. Ignore the effects of Sun and other bodies. Assume that the heights of the tides are proportional to the tidal force. If the moon is brought to half its distance from the Earth, the height of the tides will, (JEST 2001; A. P. French : tidal force is inversely proportional to r^3)

a) remain the same b) increase by a factor of 2
c) increase by a factor of 4 d) increase by a factor of 8

10. A simple pendulum of length 'l' undergoes small oscillations in the laboratory with a period T. The same pendulum oscillates in an accelerated lift with a period 2T. If the acceleration due to gravity is g, then we infer that, (JEST 2001)

a) the lift is accelerating upwards at g .
b) the lift is accelerating downwards at ½ g.
c) the lift is accelerating downwards at ¾ g. ??? it shoud be upward acceleration
d) the lift is accelerating downwards at g.

11. A particle of mass 'm' constrained to move along the X-axis, is subjected to a force, whose potential energy is given by $V(x)= a+bx+cx^2$, where a, b, and c are positive constants. If the particle is disturbed slightly from its equilibrium position, (JEST sample question; solved already)

a) it performs a simple harmonic motion with period $2\pi\sqrt{m/2c}$
b) it performs a simple harmonic motion with period $2\pi\sqrt{ma/2b^2}$
c) it moves with constant acceleration.

d) it moves with constant velocity.

12. If we reduce the units so that acceleration and angular momentum are dimensionless, which of the following quantities would have a mass dimension >1? (JEST 99)
    a) Moment of inertia b) Energy
    c) Pressure d) Velocity

13. A particle of mass 'm' evolves in a cubic potential $V(x)=ax^3$ in one dimension. A complete description of the dynamics of the particle (given its initial state) can be obtained from the equation(s) (JEST 99)
    a) $dx/dt = p/m$ b) $dx/dt = p/m$, $dp/dt = -3ax^2$
    c) $dx/dt = p/m$, $dp/dt=3ax^2$ d) $dp/dt=ax^3$

14. The Moon goes round the Earth in about 28 days. Given that the Earth-Moon distance is about 60 times the radius of the Earth, the period of a near-Earth (radius of orbit almost equal to the radius of the Earth) satellite is approximately, (JEST 99)

    a) 2 days b) 11 hours c) 90 hours d) 90 minutes

15. A particle of mass m moves under the action of a central force whose potential is $V(r) = mkr^3$, (k>0). For what kinetic energy will the orbit be a circle of radius 'a' about the origin? (JEST 99)

(Corrected)

    a) ½ mka^3 b) 3mka c) 3/2 mka^2 d) 3/2 mka^3

16. Consider a cube with sides of length **a** made of some material of density ρ , floating in a liquid of density $\rho_0 > \rho$. If we press the cube slightly and then release it so that it undergoes vertical oscillations, find the time period of such oscillations. (The acceleration due to gravity is g). (JEST 99)

    a) $2\pi\sqrt{a/g}$ b) $2\pi\sqrt{(\rho a/\rho_0 g)}$
    c) $2\pi\sqrt{(2\rho a/\rho_0 g)}$ d) $2\pi\sqrt{(\rho a/2\rho_0 g)}$

17. A rocket is fired vertically from the earth's surface and attains escape velocity. The equation governing its flight is given by (a is the radius of the earth, r the distance from the centre of the earth and " $\hat{r}$ " is the unit vector along r and " $\vec{V}$ " is the velocity of the rocket) (JEST 99)

    a) $md\vec{V}/dt = -mgr\hat{r}$ b) $md\vec{V}/dt = -(mga^3/r^3)\hat{r}$
    c) $d/dt(m\vec{V}) = (mga^2/r^2)\hat{r}$ d) $d/dt(m\vec{V}) = -mgr\hat{r}$

18. Lagrangian for a charged particle in an electromagnetic field is given by

a) $\frac{1}{2}mv^2 + q\phi + \frac{q}{c}\vec{v}.\vec{A}$ b) $\frac{1}{2}mv^2 - q\phi - \frac{q}{c}\vec{v}.\vec{A}$

c) $\frac{1}{2}mv^2 - q\phi + \frac{q}{c}\vec{v}.\vec{A}$ d) $\frac{1}{2}mv^2 + q\phi - \frac{q}{c}\vec{v}.\vec{A}$

19. In the case of elliptic orbits energy is proportional to

a) a b) a^{-1}
b) a^{-3} d) a^3

20. An artificial satellite revolves about the earth at height R above the surface, the orbital period so that a man in the satellite will be in a state of weightlessness is

a) $2\pi\sqrt{\frac{g}{R}}$ b) $2\pi\sqrt{\frac{R}{g}}$

b) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ d) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

21. The Impact parameter, s, defined as the perpendicular distance between the center of force and the incident velocity, is proportional to

a) E b)$E^{1/2}$
c) E^{-1} ? d) $E^{-1/2}$

22. In the Rutherford scattering cross-section, differential scattering cross section is proportional to (e is eccentricity?)

a) e b) e^2
c) e^3 d) e^4

23. Lorentz transformations assume

a) Space and time are both relative
b) Space is relative but time is absolute
c) Space is absolute but time is relative
d) Space and time are both absolute

24. A cube has a side l_0 when at rest. If the cube moves with velocity v parallel to its one edge, then its volume becomes

a) l_0^3 b) $l_0^3\left(1-\frac{v^2}{c^2}\right)^{-3/2}$

c) $l_0^3\left(1-\frac{v^2}{c^2}\right)$ d) $l_0^3\left(1-\frac{v^2}{c^2}\right)^{1/2}$

25. A sphere is moving along an axis parallel to x axis with a speed comparable to c. The sphere looks like

a) smaller sphere b) distorted
c) bigger sphere d) a spheroid

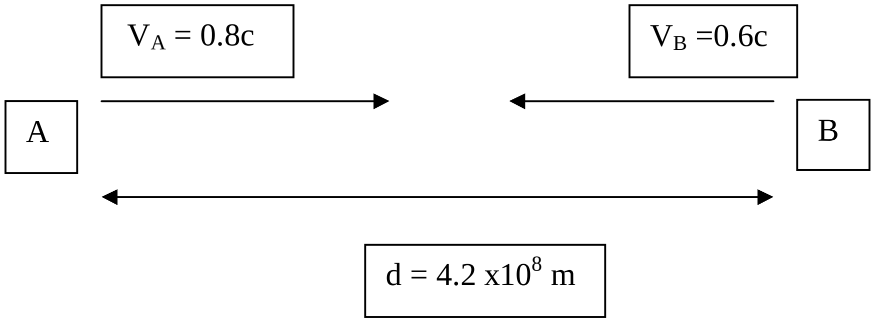
26. Possible longitudinal normal modes of the linear symmetric triatomic molecule is/are

a) One b) Two
c) Three d) Four

27. The length contraction –

e) Predicts that the length of an object approaches zero as its speed approaches the speed of light in vacuum
f) Predicts that there is no change in the length of an object when its speed approaches the speed of light in vacuum
g) Predicts that the length of an object reduces to half when its speed approaches the speed of light in vacuum
h) Predicts that the length of the object is directly proportional to its velocity

28. Two rockets A and B, in a frame of Earth based observer are moving on a collision course as shown below:



Find t_A and t_B, the times as measured by the observers on rockets A and B respectively. (JEST 99)

(a) $t_A=t_B= 1$sec. (b) $t_A= 0.6$ sec, $t_B= 0.8$ sec.
(c) $t_A=0.8$ sec, $t_B=0.6$ sec. (d) $t_A= t_B=0.6$ sec.

29. An atom moving at speed 0.3 c emits an electron along the same direction with speed 0.6c in the inertial rest frame of the atom. The speed of the electron in the lab frame is equal to

(a) 0.9 c (b) 0.51 c
(c) 0.66c (d) 0.76 c

30. Three equal masses m are rigidly connected to each other by mass less rods of length l forming an equilateral triangle. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, the ratio of kinetic energy of the assembly for an axis through one corner of the triangle compared with that for an axis through the centre of the triangle is

(a) 3 (b) 2
(c) 1 (d) 1/2

31. In a system of units in which c=1, which of the following is a Lorentz transformation? (GATE. 98)

a. $x'=4x$, $y'=y$, $z'=z$ and $t'=0.25\,t$
b. $x'=x-0.5t$, $y'=y$, $z'=z$ and $t'= t+x$
c. $x'=1.25\,x -0.75\,t$, $y'=y$, $z'=z$, $t'=0.75\,t-1.25\,x$
d. $x'=1.25\,x-0.75\,t$, $y'=y$, $z'=z$ and $t'=1.25\,t- 0.75\,x$

32. Suppose that a proton has been accelerated to have a total energy, which is 100 times its rest mass energy, estimate the difference between the proton's speed and the speed of light: (JEST 99)

(a) 3×10^{3} m/s (b)) 3×10^{4} m/s (c)) 3×10^{5} m/s (d)) 3×10^{6} m/s

33. A particle of mass m moves in two dimensions in a potential V(x, y) = (½){k(x ^{2}+ y ^{2}) + 2k' (xy)}. The normal modes of this system has frequencies (IISc. 2003)

a) $\sqrt{\frac{k}{m}}$ *and* $\sqrt{\frac{k'}{m}}$

b) $\sqrt{\frac{k+k'}{m}}$ *and* $\sqrt{\frac{k-k'}{m}}$

c) $\sqrt{\frac{k+k'}{2m}}$ *and* $\sqrt{\frac{k-k'}{2m}}$

d) $\sqrt{\frac{k}{2m}}$ *and* $\sqrt{\frac{k'}{2m}}$

34. A particle of mass m_0 has kinetic energy equal to m_0c^2, that is equal to its rest mass energy. The particle's momentum is m_0c times

(a) ½ (b) 2
(c) 1 (d) 3

35. The kinetic energy of a relativistic particle is (IISc. 97 – similar question)

a) m_0c^2 b) $E - m_0c^2$
c) $\frac{1}{2}mv^2$ d) $E + m_0c^2$

36. An electron gains energy so that its mass becomes 2 times its rest mass. Its speed is

a) $\frac{\sqrt{3}}{2}c$ b) ¾c
c) 3/2 c d) √(3/2)c

37. There are six particles lying on a plane. The degrees of freedom associated with them are

a) 6 (b) 18
(c) 12 (d) none of the above.

38. In projectile motion, the velocity is perpendicular to the acceleration

a) all instants (b) two instants
(c) one instant (d) none of these.

39. If two particles move in opposite directions, each with half the speed of light c magnitude of the relative velocity between them is,

i) 0.8 c (b) c
(c) $c^2/2$ (d) 0.5c

40. The transformation Q = p, P= -q is canonical . It is

j) True (b) false.

41. A particle of mass M, initially at rest decays into two particles with rest masses m_1 and m_2. The total energy of m_1 is

(a) $c^2\left(M^2 + m_1^2 - m_2^2\right)/2M$
b) $c^2\left(M^2 + m_1^2 - m_2^2\right)/M$
c) $c^2\left(M^2 - m_1^2 + m_2^2\right)/2M$
d) $c^2\left(M^2 + m_1^2 + m_2^2\right)/2M$

42. The principle of equivalence in the special theory of relativity is true for

a) any two inertial frames of reference
b) any two non-inertial frames of reference
c) any two frames of reference
d) any two frames of references, of which at least one is inertial

43. For repulsive inverse square forces, the shape of the orbit is

k) elliptic, (b) parabolic
(c) hyperbolic (d) all of the above.

44. In Galelian transformations, time interval is

l) different for different inertial frames
m) is a vector
n) is a relative concept
o) is same for all inertial frames.

45. A body of mass M is moving away from the origin, with a constant velocity, along a line parallel to the X-axis. Its angular momentum with respect to the origin

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing.

46. If a generalized coordinate has the dimension of momentum, the generalized velocity will have the dimension of

a) Velocity b) Acceleration
c) Force d) Torque

47. For an electrical circuit comprising an inductance L and capacitance C, charged to q coulombs and the current flowing in the circuit is i amperes, Lagrangian can be represented as

a) $L\dot{q}^2 - \frac{q^2}{C}$ b) $\frac{1}{2}L\dot{q}^2 - \frac{1}{2}q^2C$
c) $\frac{1}{2}L\dot{q}^2 - \frac{1}{2}\frac{q^2}{C}$ d)) $\frac{1}{2}L\dot{q}^2 + \frac{1}{2}q^2C$

48. Lagrangian for a compound pendulum is

a) $\frac{1}{2}I\dot{\theta}^2 - mgl\cos\theta$
b) $\frac{1}{2}I\dot{\theta}^2 + mgl\cos\theta$
c) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}I\dot{\theta}^2 - mgl\cos\theta$
d) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}I\dot{\theta}^2 + mgl\cos\theta$

49. If a coordinate corresponding to a rotation is cyclic, rotation of the system about given axis remains invariant. Then the following quantity is conserved.

a) Linear momentum b) Angular momentum
c) Kinetic energy d) Potential energy

50. Hamilton's canonical equations of motion are

a) $\dot{q}_i = \frac{\partial H}{\partial p_i}$ and $\dot{p}_i = \frac{\partial H}{\partial q_i}$

b) $\dot{q}_i = \frac{\partial H}{\partial p_i}$ and $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

c) $q_i = \frac{\partial H}{\partial \dot{p}_i}$ and $p_i = \frac{\partial H}{\partial \dot{q}_i}$

d) $q_i = \frac{\partial H}{\partial \dot{p}_i}$ and $p_i = -\frac{\partial H}{\partial \dot{q}_i}$

51. If a coordinate is cyclic, Hamiltonian would reduce the number of variables in the new formulation by

a) One
b) Two
c) Three
d) Four

52. The operator which represents the two variables should commute if the Poisson brackets of the two variables have the value

a) 1
b) 0
c) $i\hbar$
d) $-i\hbar$

53. The force that is always directed away or towards the force centre and the magnitude of which is a function only of the distance from the fixed centre is known as

a) Coriolis force
b) Centripetal force
c) Centrifugal force
d) Central force

54. For attractive inverse square forces, the shape of the orbit is

a) Elliptic
b) Parabolic
c) Hyperbolic
d) All of the above

55. A ball is released from rest from a great height above the ground in Delhi. It will hit the ground

a) Exactly below the point of release
b) Slightly east of the vertical
c) Slightly west of the vertical
d) Silghtly north of the vertical

56. A Particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. The equation of motion governing the trajectory is $r = a\cos\theta$. The force is then inversely proportional to

a) r^2
b) r^3
c) r^4
d) r^5

57. At the instant of collision of two balls

a) Kinetic energy increases
b) Potential energy decreases
c) Part of the kinetic energy is changed in to heat ?
d) The Hamiltonian remains conserved.

58. The Lagrangian of a particle has the form

$L = \frac{1}{2}mr^2\theta^2 + \frac{1}{2}mr^2 \sin^2\theta\dot{\varphi}^2 - mgr\cos\theta$ where r, θ and φ are spherical polar coordinates. Which of the following statements is true?

a) r, θ and φ and do not depend on time.
b) φ is a cyclic coordinate
c) θ is a cyclic coordinate
d) r is a cyclic coordinate

59. For a charged particle in an electromagnetic field, the canonical moments are

a) $mv + \frac{q}{c}A$
b) $\frac{1}{2}mv^2 + \frac{q}{c}A$
c) $mv - \frac{q}{c}A$
d) $\frac{1}{2}mv^2 - \frac{q}{c}A$

60. A particle of mass m moves along a straight line and is attracted towards a point on this line with a force proportional to the distance x from that point. The Lagrangian of the system is

a) $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$
b)) $\frac{1}{2}mv^2 - \frac{1}{2}kx^2$
c)) $\frac{1}{2}mv^2 + kx^2$
d)) $mv^2 + \frac{1}{2}kx^2$

61. The second law of Kepler for planetary motion is a consequence of the law of conservation of

a) Isospin
b) Energy
c) Linear momentum
d) Angular momentum

62. How many degrees of freedom a rigid body possesses?

a) 3
b) 6

c) 9 d) infinite

63. When a rigid body rotates about a given axis, the degrees of freedom it will have is

a) 1 b) 2
c) 3 d) 4

64. For a rectangular coordinate system, the inertia tensor is given by

$$I = \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$ The principle moments of inertia are

a) λ, 3λ, λ b) λ, -λ, 5λ
c) λ, 2λ, 3λ d) 0, λ, 6λ

65. A planet has a period of revolution about the sun equal to T and the mean distance from then sun equal to R. T^2 varies directly as

a) R b)R^2
c) R^3 d) R^4

66. For a particle (m=mass, l=angular momenta, r = distance from a fixed point) attracted by an inverse square law, central force is $f = -k/r^2$. By applying the conservation of energy principle, we can say that

a) $\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{K}{r} = const$ b)) $\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{K}{r} = const$
c) $\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{K}{r} + \frac{l^2}{2mr^2} = const$
d) None of the above

67. At a height equal to Earth's radius, above the Earth's surface, the acceleration due to gravity is

a) g b) g/2
c) g/4 ?? d) g/8

68. A xyz coordinate system, initially coinciding with an inertial frame xyz, rotates with an angular velocity $\vec{\Omega} = 2\hat{i} + t^2\hat{j} + (2t+4)\hat{k}$ where t= time. The position vector of the particle at time t in (xyz) is given by $\vec{r} = (t^2+1)\hat{i} - 6t\hat{j} + 4t^3\hat{k}$. The apparent velocity at time =1 s is given by

a) $v' = 2\hat{i} + 6\hat{j} + 4\hat{k}$
b) $v' = 2\hat{i} - 6\hat{j} + 12\hat{k}$
c)) $v' = 2\hat{i} + 6\hat{j} + 12\hat{k}$
d) $v' = 2\hat{i} + 6\hat{j} - 4\hat{k}$

69. The slope of PE versus position graph represents

a) Force
b) Work
c) Power
d) Momentum

70. Potential energy U(r) corresponding to the central force $F = \frac{K}{r^2}$ would be

a) –K/r (CORRECTED)
b) Kr
c) $-K/r^2$
d) K/r^2

1. For an electrical circuit comprising of an inductance L and capacitance C, charged to q coulombs and the current flowing in the circuit is I amperes, write down the Lagrangian. Arrive at the equations of motion.

2. Write down the Lagrangian for a charged particle in the electromagnetic field. Arrive at the canonical momenta. Also write down the Lagrangian for static electric field and static magnetic field.

3. A bead is sliding on a uniformly rotating wire in a force free space. Arrive at the equations of motion.

4. Write down the Lagrangian for a particle moving under the influence of central force.

5. A particle m describes a circular orbit under the influence of a central force $V(r)=kmr^3$ (k>0) directed toward a point on the circle. Prove that $F(r) \propto 1/r^5$. Derive the expression for the energy for which the orbit will be a circle of radius a. Also derive the period of the circular motion.

6. Prove that $Q = \sqrt{(2q)}e^{a} \cos p$ and $P = \sqrt{(2q)}e^{-a} \sin p$ is a canonical transformation

7. For what values of α and β, $Q = q^{\alpha} \cos \beta p$ and , $P = q^{\alpha} \sin \beta p$ represents a canonical transformation.

8. A charged particle is moving under the influence of a point nucleus. Find the orbit of the particle and periodic time in the case of an elliptic orbit.

9. Write down the Hamilton's equation of motion fro a charged particle in a electromagnetic field.

10. A mass of m, moves in a circular orbit of radius r_0 under the influence of a central force whose potential is $-k/r^{n}$. Show that the circular orbit is stable under small oscillations if n<2.

JEST 2001 (38), JEST 2000 (5),